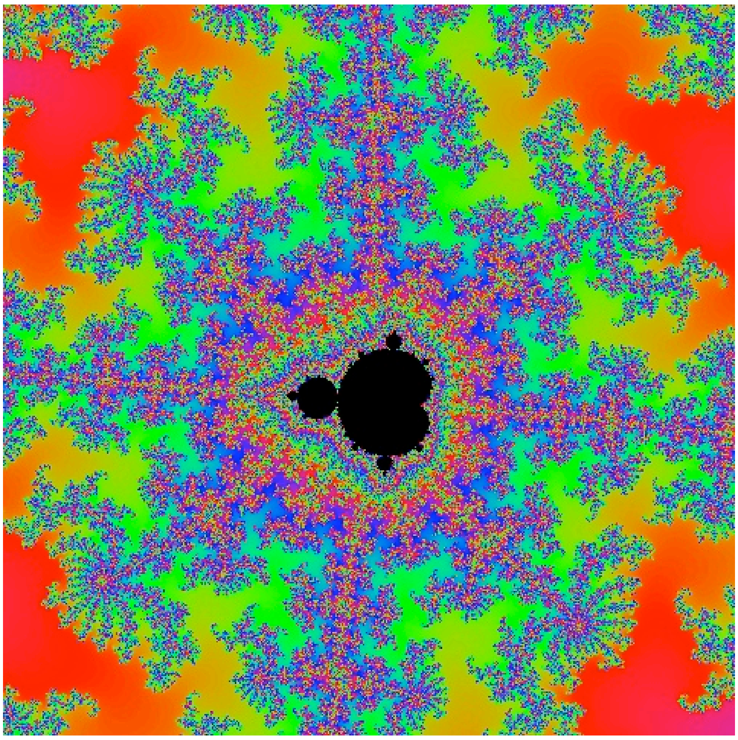

MATHS 745
Chaos, Fractals and Bifurcations

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Chaos, fractals and bifurcation, and their application to wide areas including commerce, medicine, biological and physical sciences. The course focuses on discrete iterations, including the classical fractals of computer science and art such as Julia and Mandelbrot sets, iterated function systems and higher-dimensional strange attractors. Quantum chaos and complexity theory are emerging frontier areas discussed in the course.

The aim is to provide a course in non-linear discrete dynamics and it's applications to many fields which will give graduate students, in addition to a knowledge of chaotic and fractal dynamics, opportunities to apply their mathematical expertise to a variety of areas which may also provide research and employment opportunities in other disciplines.

A core of lecture material on the central ideas lead into applications to a variety of sciences and some areas of economics and the humanities. The aim is for this to be complemented at the end by a series of short seminar talks by the class members about a mini-project in a related area.

Assessment:

One class test 50%, two assignments 10% each, , including one or two computer simulations of a dynamical system, a mini-project on any related area you find interesting 15%, which will also be the subject of a 20 minute talk 15%.

Room Bookings:
M, W, F 4pm.

Course Outline:

(a) Introduction and Motivating Examples
Axioms of chaos, symbolic dynamics. Examples of chaotic and fractal systems.

(b) Iterations Bifurcations and the Development of Chaos
The logistic map and its properties including period doubling, the tangent bifurcation.
Feigenbaum numbers, crises, intermittency, topological conjugacy, Sarkovski's theorem etc.
The three classical routes to chaos. The circle map, mode-locking and the devil's staircase.
Structural stability, bifurcation theory, Morse-smale systems homoclinic points, kneading.

(c) Fractals. Metric spaces and affine mappings. Code space. The space H(X). Iterated function systems. Fixed points. Random and deterministic algorithms. The fractal basis of natural forms. Complex iterations. Computer methods for Julia sets and the Mandelbrot set. Complex analytic dynamics, normal families and exceptional points, the geometry of Julia sets.

(d) Continuous Flows and Higher Dimensional Systems The Lorentz & R?ssler flows, Henon-Heiles and double scroll system. The dynamics of linear maps. Attractors: The Smale horseshoe and the solenoid. Stable and unstable manifolds. The Henon map. Conservative flows, elliptic and hyperbolic points and cantori. Homoclinic & heteroclinic points

(e) Measures of Chaos and Time-series Analysis Hausdsorff & correlation dimensions, Liapunov exponent, Kolomogrov entropy and information, the power spectrum. Takens theorem and time series.

(f) Applications of Chaos and Bifurcations to physics, chemistry, biology, medicine, geography, and economics. Chaos in the electroencephalogram, chemical oscillations, heartbeats, land forms, crustal movements, astronomy.

(g) Complex systems, Symmetry, and the Quantum limit Transition to chaos, anti-chaos. Complex systems, digital systems with a mixing parameter. Symmetric chaos. Representations of quantum chaos.

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