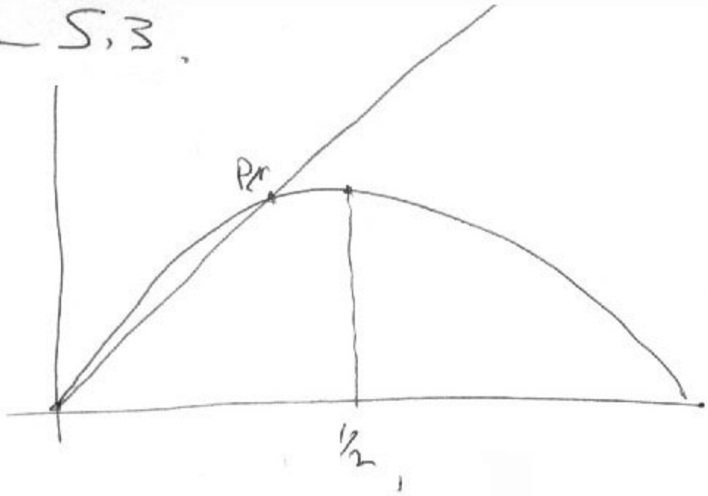
Proposition 5.3.



$1 < \mu < 2$. So $p_\mu = \frac{\mu - 1}{\mu}$ $\quad 0 < p_\mu < \frac{1}{2}$

$x \in [0, p_\mu]$ $\quad f(x) > x$ and. $f'(x) > 0$ on $[0, \frac{1}{2}]$

so $\quad x < p_\mu \Rightarrow f(x) < f(p_\mu) = p_\mu$.

$x \in [p_\mu, \frac{1}{2}]$ $\quad f(x) < x$

$x > p_\mu \Rightarrow f(x) > f(p_\mu) = p_\mu$

So. $|f(x) - p_\mu| < |x - p_\mu|$

Also $f(\frac{1}{2}) \leq 2\frac{1}{2}(1 - \frac{1}{2}) = \frac{1}{2}$ and $f(x) \leq f(\frac{1}{2})$

so $\quad x \in [\frac{1}{2}, 1]$ $\quad f(x) \in [0, \frac{1}{2}]$

$\mu = 2$. $\quad f(x) = 2x(1 - x)$

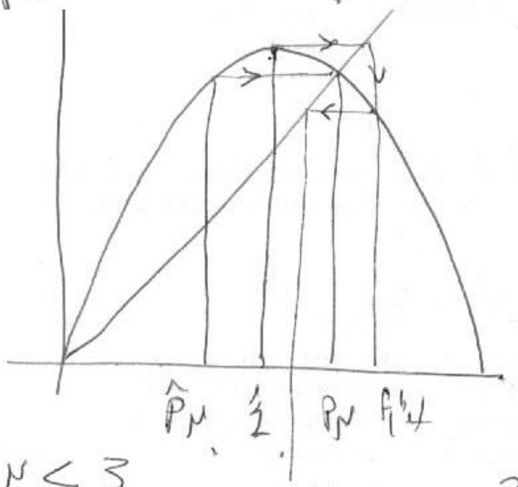
$p_\mu = \frac{2-1}{2} = \frac{1}{2}$.

$x \in [0, \frac{1}{2}]$ $\quad f(x) > x$. $\quad x < \frac{1}{2} = p_\mu \Rightarrow f(x) - f(\frac{1}{2}) = \frac{1}{2}$

since $f'(x) > 0$ on $(0, \frac{1}{2})$

also. $x \in [\frac{1}{2}, 1] \Rightarrow f(x) \in [0, \frac{1}{2}]$

Proposition 5.3.



$$x = \mu x - \mu x^2.$$
$$x\left(\frac{\mu-1}{\mu} - x\right) = 0$$
$$P_\mu = \frac{\mu-1}{\mu}.$$

$2 < \mu < 3$

$f(\frac12) = \mu\cdot\frac12(1-\frac12) = \mu/4.$

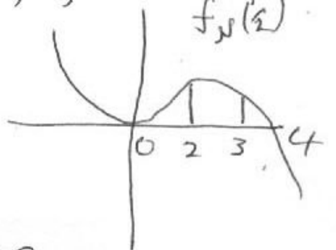
$2<\mu<3$ so $\frac12 < \mu/4 < 3/4$

$[\hat{P}_\mu\ P_\mu]$ is mapped by f to $[P_\mu, \mu/4]$

Now $f_\mu^2(\frac12) = \mu\ \frac{\mu}{4}\cdot\left(1-\frac{\mu}{4}\right) = \frac{\mu^2}{4}\left(1-\frac{\mu}{4}\right)$

This is a cubic with roots $0, 0, 4$.

It is concave down for $\mu > 0$



$f_2^2(\frac12) = \frac44(1-\frac12) = \frac12.$

$f_3^2(\frac12) = \frac94(1-\frac34) = \frac{9}{16} > \frac12.$

$f(\frac12) = \mu/4 > \frac{\mu-1}{\mu}$ on $2<\mu<3$ since

$\mu^2 - 4\mu + 1 = (\mu-2)^2 \geq 0$ $(>0\ 2<\mu<3)$

Hence $f^2(\frac12) < \frac{\mu-1}{\mu}$ since $f'(x) < 0$ on $[P_\mu, 1]$

Hence. $f^2[\hat{P}_\mu, P_\mu] = f[P_\mu\ \frac{\mu}{4}] \in [\frac12\ P_\mu]$.

Now $|f_\mu'(x)| < 1$ on $[\frac12\ P_\mu]$, so f is attracting to P_μ on $[\frac12\ P_\mu]$

<u>Check:</u> $f_\mu'(x) = |\mu - 2\mu x| = \mu|1-2x| = \mu\left|\frac{\mu-2\mu+2}{\mu}\right|$ $x = \frac{\mu-1}{\mu}$

$= |2-\mu|$

$-1 > 2-\mu > 0$

On $(0\ \hat{P}_\mu]$ $f(x) > x$ so $\exists k > 0$

$\therefore f^k(x) \in [\hat{P}_\mu\ P_\mu]$.

On $[P_\mu\ 1]$, $f(x) \in (0\ P_\mu]$, so

all points x in $(0,1) \xrightarrow[f_\mu]{} P_\mu$