$$n! = \sqrt{2\pi}\, n^{n+1/2} e^{-n} \left(1 + O(1/n)\right).$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{\sqrt{2\pi}\; n^n n^{1/2} e^{-n}}{\sqrt{2\pi}\; k^k k^{1/2} e^{-k} \sqrt{2\pi}\,(n-k)^{n-k} e^{-(n-k)}}$$

$$= \sqrt{\frac{n}{2\pi k(n-k)}} \quad \frac{n^n}{k^k (n-k)^{n-k}}$$

$$= \quad 〃 \quad \left(\frac{n}{k}\right)^k \left(\frac{n}{n-k}\right)^{n-k}$$

$$= \quad 〃 \quad \left(\frac{k}{n}\right)^{-k} \left(1 - \frac{k}{n}\right)^{-(n-k)}$$

$$= \left[2^{\log_2\left(\frac{k}{n}\right)}\right]^{-k} \cdot \left[2^{\log_2\left(\frac{n-k}{n}\right)}\right]^{-n-k}$$

$$= 2^{n\left(-\frac{k}{n}\log_2\left(\frac{k}{n}\right) - \frac{n-k}{n}\log_2\left(\frac{n-k}{n}\right)\right)}$$

$$= 2^{nH(k/n)}$$

$$H(\xi) = -\xi \log_2(\xi) - (1-\xi)\log_2(1-\xi).$$

$$\tilde{f}(\xi) = D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)}$$

$$\left(\frac{1}{2}\right)^n = \varepsilon$$

$$r = \frac{1}{2}$$

$$= \lim_{n \to \infty} \frac{\log \binom{n}{k}}{\log \left(\frac{1}{\frac{1}{2}}\right)^n}$$

$$= \lim_{n \to \infty} \frac{\log \left(2^{n H\left(\frac{k}{n}\right)}\right)}{\log_2 2^n}$$

$$= H\left(\frac{k}{n}\right) = H(\xi) = -\xi \log_2(\xi) - (1-\xi)\log_2(1-\xi)$$

$$p^k (1-p)^{(n-k)} r^{-n\alpha\left(\frac{k}{n}\right)} = 1$$

$$\alpha\left(\frac{k}{n}\right) = \frac{\frac{k}{n}\log p + \left(1-\frac{k}{n}\right)\log(1-p)}{\log r}$$

$$f(\alpha) = \tilde{f}(\xi(\alpha))$$

$$\frac{df}{d\alpha} = \frac{d\tilde{f}}{d\xi} \cdot \frac{d\xi}{d\alpha}$$

$$\frac{d\tilde{f}}{d\xi} = -\log\xi - \xi\frac{1}{\xi} + \log(1-\xi) + (1-\xi)\frac{1}{(1-\xi)}$$

$$= -\log\xi + \log(1-\xi)$$

$$= \frac{\log\xi - \log(1-\xi)}{\log p - \log(1-p)}$$

$$\frac{d\alpha}{d\xi} = \log p - \log(1-p)$$

$$p^k (1-p)^{n-k} \cdot r^{-n\alpha(\xi)} = 1$$

$$k \log p + (n-k) \log(1-p) - n\alpha(\xi) \log r = 0$$

$$\frac{k}{n} \log p + \left(1 - \frac{k}{n}\right) \log(1-p) = \alpha(\xi) \log r.$$

$$\alpha(\xi) = \frac{\xi \log p + (1-\xi) \log(1-p)}{\log r.}$$

$$D_q = \lim_{r \to 0.} \frac{S_q}{\log r.}$$

$$D_0 = \lim_{r \to 0} \frac{\log \sum_1^N p_i^{\,0}}{\log r} \left(= \frac{1}{0-1} \right) = -\lim_{r \to 0.} \frac{\log N}{\log r}$$

$$= \lim_{r \to 0} \frac{\log N}{\log(1/r)}$$

$$S_1 = \lim_{q \to 1} \frac{\log \sum_{i=1}^N p_i^{\,q}}{q-1}$$

$$= \lim_{q \to 1} \frac{f'}{g'} = \frac{\frac{1}{\sum p_i^q} \sum (\log p_i) p_i^{\,q}.}{1}$$

$$= \frac{1}{\sum p_i} \sum (\log p_i) p_i = \sum (\log p_i) p_i \quad \left(\sum p_i = 1 \right)$$

$$\lim_{n \to \infty} \sum_{i=1}^{N} p_i^q r^\tau = 1$$

$$r^\tau \sum_{i=1}^{n} p_i^q = 1$$

$$\tau \log r + \log \sum_{i=1}^{N} p_i^q = 0$$

$$\tau = \frac{-\log \sum_{i=1}^{N} p_i^q}{\log r}.$$