445.745 Assignment 2

Please hand in 14th May 2008

1.[25] (a) Use the notes from "The Science of Fractal Images" to implement the continuous potential method outside the Mandelbrot set of the complex logistic function $f(z) = r.z.(1\text{-}z)$. A Matlab m-file performing the level set method on the standard Mandelbrot set is provided on the 745 web site. Print out your image with a copy of the listing.

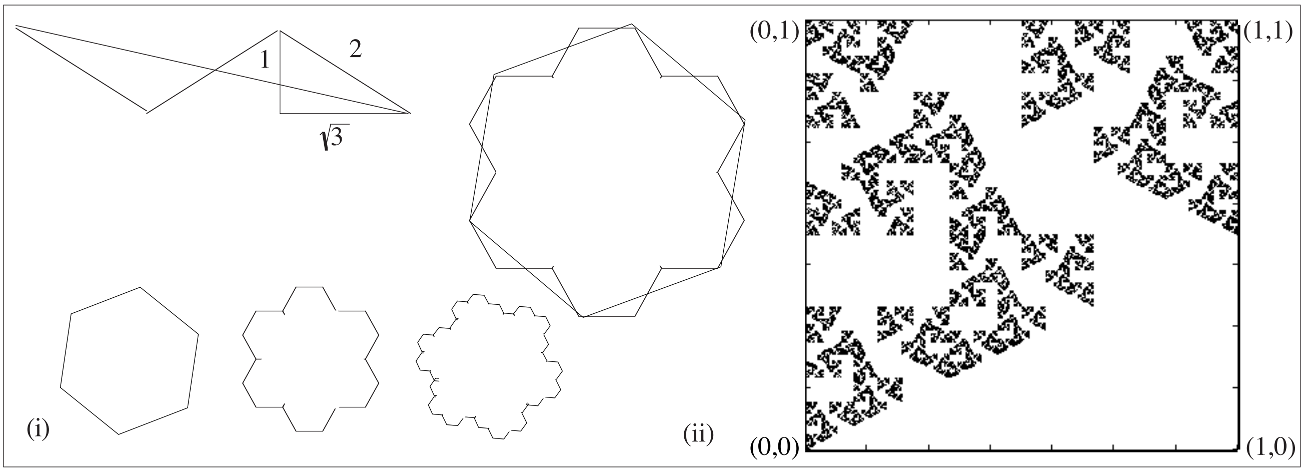
2.[15] For the real quadratic $f(x) = r.x.(1\text{-}x)$:

(a) Find a value of r such that f has a superattracting period 2 orbit.

(b) Find the two x values which form this orbit and verify that f maps each to the other.

Hint: A superattracting period 2 orbit of f must have $f^{(2)}$ with superattracting fixed points.
These must have derivative zero to be superattracting, so they must also be relative extrema.
By symmetry one of these must be $x = 1/2$, so we can solve $f^{(2)}(1/2)=1/2$ for r.
By inspection $r = 2$ is a root, but this is a period 1 superattrractor of f.
Dividing the cubic in r by this root gives a quadratic, one of whose roots is the one.
You can represent the answers to (a) and (b) exactly in terms of square roots.

3.[10] Determine the fractal dimension of the Peano set P illustrated in (i) below, by counting numbers and lengths of edges of a pair of polygons in a sequence converging to P. Notice this is an example of a fractal whose enclosed area remains constant but the length of its 'coastline' tends to infinity. One can also generate its fractal dimension by considering its internal symmetries which convert a hexagon into 7 comprising the same area.



4.[15] The fractal shown in (ii) above is defined by an iterated function system consisting of four affine mappings $F_i(x,y)$ on [0,1]x[0,1] with two different contractivity factors.

Determine from the internal symmetries the coefficients of the four affine mappings

$$F_i(x,y) = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}.$$

Compute the Matlab random iteration algorithm on the web site to check your answer is correct.

5.[10] Verify that the fractal dimension of the IFS in 4 as the unique solution to

$\sum_{n=1}^{N} | s_n |^D = 1$ where s_n are the contractivity factors of each w_n in the IFS is approximately 1.605.

6.[25] Adapt the IFS m-file provided on the 745 web page to iterate the Chirikov map:

$$p_{n+1} = p_n + K\sin(\theta_n),\ \ \theta_{n+1} = \theta_n + p_{n+1}\ ,\ \text{for } K=0.98,$$

which forms a model of the discrete integral of a sinusoidally kicked rotator (circle map).

You will need to use a grid of starting positions between 0 and 2π in both variables and keep the variables in this range, by adding or subtracting 2π if they are out of range, to avoid a memory error. Your output should look like Fig 6(a) in the brain and chaos paper.