11. MAGIC POLYGONS

1. Introduction

In these pages I will try to show how regular polygons can be used for an interesting way to practice multiplications of numbers up to one hundred while at the same time learning something about the rotations and symmetries of regular polygons. To each rotation and to each reflection of a particular regular polygon, will correspond a certain multiplication, always by the same number for each of these geometrical transformations. I will give a detailed description of how to make such a “magic polygon” in the case of the equilateral triangle, the square, the pentagon and the hexagon, with the numbers that need to be written on the faces of each of these polygons. The numbers will be in tables from the heptagon to the decagon.

2. The Equilateral Triangle

One face of the triangle should look like Figure 1a. The multiplications should be understood in modulo 9. This means that from any product greater than 8, any 9 or multiple of 9 must be subtracted. What is “left over” must be considered the value of the product.

On the reverse side of the triangle there should be an circled 2 at the back of the circled 1, with a X 5 written underneath the circled 2, in the same way the X 2 is written underneath the circled 1. At the back of the circled 4 and X 8, there should be a circled 5 with a X 8 written underneath it, and at the back of the circled 7, there should be a circled 8 with a X 2 written underneath it.

The circular arrow on the reverse side should point counter clockwise and should be allocated the multiplication X 7.

The circled numbers are the multiplicands and the numbers preceded by an X are the multipliers.

When you have prepared your triangle with all the above written on it, you will note that by turning through one third of a whole turn in the direction of the arrow, each circled number is replaced by its multiple shown at the circular arrow, with of course any 9's being subtracted. If you rotate against the arrow, each number will be multiplied by the multiplier which is written at the centre of the opposite face If, on the other hand, you “flip” your triangle, holding it firmly at each end of an axis of symmetry, each circled number will get multiplied by the multiplier written on your chosen axis when you have put your triangle down again with the other face showing.

The above can be shown more systematically as follows:

Numbers	1	4	7	any number			
Multipliers	X 2	X 8	X 5	clockwise	X 4	counter	X 7

Numbers	2	5	8	any number			
Multipliers	X 5	X 8	X 2	counter	X 7	clockwise	X 4

The numbers in the lower part of a column should be back to back to the numbers in the upper part.

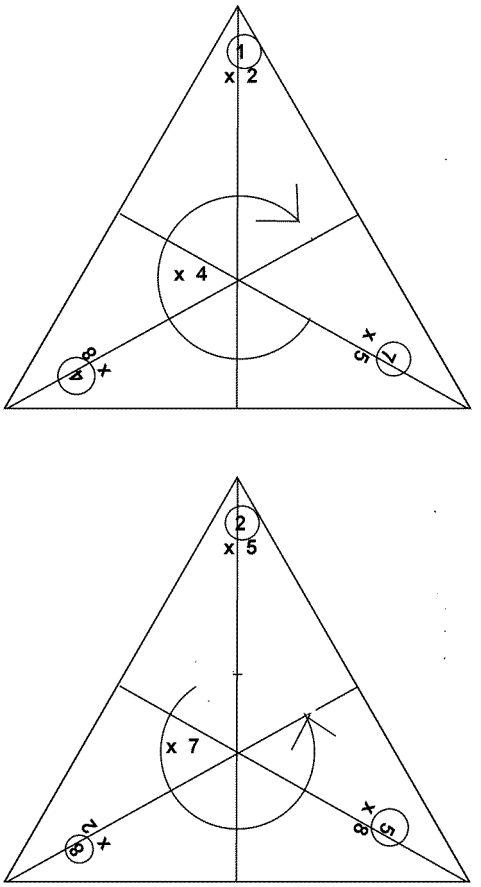

Figure 1. The two faces of the triangle.

You will notice that the circled numbers increase by 3 each time you turn the triangle through a third of a turn following the arrow (again subtracting 9 if needed!). If you turn the triangle against the arrow, the numbers increase by 6 for each turn.

Another thing you might have noticed is that if you calculate the product of two circled numbers that are back to back to each other, you always obtain 2 as the product (after subtracting enough 9's). This happens because we have used a times 2 multiplier to stand under the circled 1. You will find similar things happening in the case of other polygons.

3. The Square

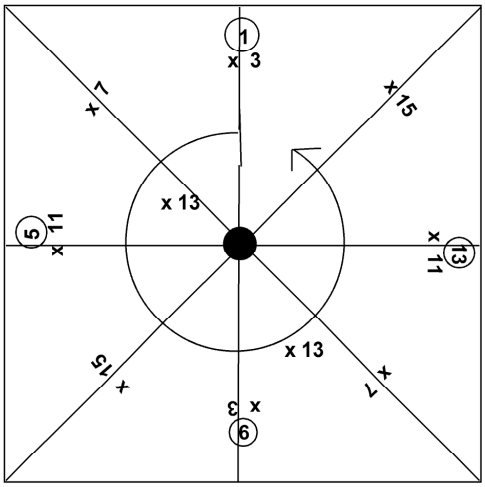

Figure 2. Face 1 of the square.

For the square there will be eight circled numbers, four on each face. They will be written near the midpoints of the sides. Under each circled number there will be a multiplier, referring to what happens if we "flip" the square about the axis of symmetry on which the multiplier is written. There will also be multipliers written on the diagonals. Here is the way the numbers must be placed, writing each number to the left of the previous number, in other words turning the square clockwise in between writing numbers:

Face 1	side	diag	side	diag	side	diag	side	diag
Number	1		5		9		13	
X	3	7	11	15	3	7	11	15

Face 2	side	diag	side	diag	side	diag	side	diag
Number	3		7		11		15	
X	11	7	3	15	11	7	3	15

The numbers in the same column must be written back to back to each other, the upper ones on Face 1 and the lower ones on Face 2. This means a counter clockwise turn for Face 2 between consecutive numbers in the table.