5. MATHEMATICAL FUN WITHOUT NUMBERS, LETTERS, FORMULAE OR EQUATIONS: PART IV
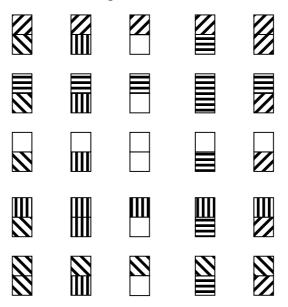
Here are some strips you can use for playing with five colours. You can choose your own colours; I shall just represent them by means of the following patterns:

We can use two such strips for sliding one along the other for making sequences. For example we could do a five-slide like this:

We could construct our village as shown below:

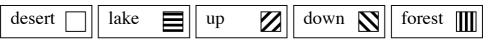How are we going to "add" and "swap" these patterns?

Of course the sky is the limit to all the possible ways of making up these rules, but let me suggest some that might come in useful in describing the rules which generate sequences.

Imagine a landscape in a remote wild place, and map out a circular tour of some of the interesting features of the place. There is a desert, by which there is a lake. On the other side of the lake some mountains rise to quite a height Starting at the foothills there is a big forest that reaches right back to the desert So we can plan our tour in the following way:

   (a) We make our way through the desert and arrive at the lake.

(b) We sail across the lake in a local fishing boat.

(c) We climb up to the top of one of the mountains.

(d) We descend, reaching the forest on the other side of the mountain.

(e) We trudge through the forest, finally reaching the desert again.

If we feel like it, we can go on and continue, doing the round trip several times over. We can use the following symbolic pictures for the various parts of our tour:

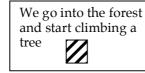| desert ☐	lake ▤	up ◩	down ◪	forest ⦚

These are my suggested rules for "adding":

(a) If two patterns to be added contain lake, then the other pattern is advanced by one "notch" along the tour to make the sum.

(b) If two patterns to be added contain forest, then the other pattern is retrogressed one notch on the tour to make the sum.

(c) If two patterns to be added contain desert, then the sum is the other pattern.

(d) Up added to down as well as lake added to forest yield the sum desert.

(e) Up added to up is forest, down added to down is lake.

It is perhaps of interest to note that adding any two successive houses in the sequence, using the above rules, we obtain as the sum the house after the next after the second of the pair of houses being considered. So given three consecutive houses of the sequence, we can construct all the rest of the 24 house sequence.

For the "swapping" operations we shall need to make up some different stories. In one of them we can go climbing trees in the forest.
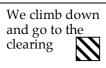
So we could, for example, use the following "story" ad call it the climbing operation:
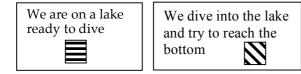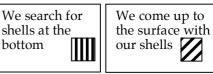
We are in a clearing within the forest ▤	We go into the forest and start climbing a tree ◩	We get to the top of the tree ⦚	We climb down and go to the clearing ◪

Another "story" could be called diving, which could go something like this:

We are on a lake ready to dive ▤	We dive into the lake and try to reach the bottom ◪	We search for shells at the bottom ⦚	We come up to the surface with our shells ◩

And yet another operation could be called opposites which would exchange the level lines with the vertical lines, as well as the up-lines with the down-lines. You can invent your own story for these, if you like!

So to sum up, we have the following sequences of patterns arising from the above operations:

Climbing	≣ → ▨ → Ⅲ → ◹ → ≣
Diving	≣ → ◹ → Ⅲ → ▨ → ≣
Opposites	≣ ◆ Ⅲ ◹ ◆ ▨

Applying any of the above operations to the desert will result again in the desert. Now let me see if I can "explain" how our sequence develops, using the above operations and the adding.

(a) To obtain the lower section of the next house, just use diving to the upper section of the house you are looking at.

(b) To obtain the upper section of the next house, find the opposite of the lower section of the house you are looking at and add this to the dived version of the upper section of the house you are looking at.

For example try ◹. If we do a dive on the upper section, we get Ⅲ .

So this will be the lower section of the next house, which it is, as we can check on the sequence.

The opposite of ◹ is ▨ and the dive of ◹ is Ⅲ .

Now adding ▨ to Ⅲ we get ≣ which is in fact the upper section of the next house**.

You could try another sequence, for example

To get the upper section of a house from the previous house, just do a climb on the lower section of the previous house.

To get the lower section of a house from the previous house, just add the upper section to the dived lower section of the previous house.

Make sure you understand how the above two sequences work and then make some of your own by sliding the strips differently and try to find the rules which will generate the sequences you obtain.