ON THE INFINITUDE OF PRIMES

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In this note we would like to offer an elementary "topological" proof of the infinitude of the prime numbers. We introduce a topology into the space of integers S, by using the arithmetic progressions (from $-\infty$ to $+\infty$) as a basis. It is not difficult to verify that this actually yields a topological space. In fact under this topology, S may be shown to be normal and hence metrizable. Each arithmetic progression is closed as well as open, since its complement is the union of other arithmetic progressions (having the same difference). As a result the union of any finite number of arithmetic progressions is closed. Consider now the set $A = \cup A_p$, where A_p consists of all multiples of p, and p runs through the set of primes ≥ 2. The only numbers not belonging to A are -1 and 1, and since the set $\{-1, 1\}$ is clearly not an open set, A cannot be closed. Hence A is not a finite union of closed sets which proves that there are an infinity of primes.

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