Symmetries of Discrete Objects

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Abstracts

Keynote talks

Automorphism groups of Beauville surfaces

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A Beauville surface is a complex algebraic surface formed from a pair of regular hypermaps of hyperbolic type with the same automorphism group G acting freely on their product. I will show that its automorphism group A has a normal subgroup I isomorphic to the centre of G, with A/I embedded in the wreath product $S_3 \wr S_2$. Using an idea of Conder and Lucchini's work on generators of special linear groups, I will show that every finite abelian group can arise as I. I will also describe joint work with González-Diez and Torres-Teigell on the action of $\mathrm{Gal}\,\overline{\mathbb{Q}}/\mathbb{Q}$ on Beauville surfaces.

Highly symmetric hyperbolic polytopes

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After a brief presentation of the vector space model for hyperbolic space, we discuss essential elements of hyperbolic polyhedral geometry such as existence, simple constructions, symmetry and regularity. The connection with fundamental polytopes of discrete groups of hyperbolic isometries allows to deal with questions such as "which discrete groups acting by isometries on hyperbolic space are of minimal co-volume?" We shall present some new results and developments in higher dimensions where highly symmetric polytopes again play a crucial role. This is joint work with Vincent Emery.

Orbits of linear groups

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I will present various results concerning the orbit sizes of finite linear groups $G \leq GL_n(F)$ (F a field) on the set of vectors. I will also discuss some applications of these to representation theory and permutation group theory.

Computing the symmetries of combinatorial objects

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We survey recent advances in the art of computing automorphisms, with special focus on the automorphism groups of graphs. A new algorithm by Adolfo Piperno will be featured.

Abstract polytopes: regular, semiregular and chiral

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An abstract n-polytope $\mathcal{P}$ is a natural generalization of the face lattice of a convex n-polytope. (You can safely think of a finite 3-polytope as a map on a compact surface.) Prompted by the classical situation, we say that $\mathcal{P}$ is *regular* if its automorphism group $\Gamma(\mathcal{P})$ acts transitively on flags in $\mathcal{P}$. Notions like *semiregularity* and *chirality* are similarly motivated.

After a brief tour of these ideas, fueled by examples, I will disuss recent work with Egon Schulte concerning the construction of *alternating* semiregular polytopes. So suppose I give you a fixed integer $k \geq 2$ and unlimited copies of two regular n-polytopes, say $\mathcal{P}$ and $\mathcal{Q}$, having matching (i.e. isomorphic) facets. Starting with a single $\mathcal{P}$, attach a copy of $\mathcal{Q}$ to each $\mathcal{P}$-facet, then a copy of $\mathcal{P}$ to each remaining 'exposed' facet of a $\mathcal{Q}$, and so on. Can this be done so that the resulting 'complex' $\mathcal{S}$ closes up with k $\mathcal{P}$'s and k $\mathcal{Q}$'s alternating around each $(n-2)$-face? If so, is $\mathcal{S}$ an $(n+1)$–polytope, and what is its automorphism group? (We do not have full answers to all such questions!)

A census of cubic vertex-transitive graphs

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We explain how some of our recent results have allowed us to compute a census of all cubic vertex-transitive graphs of order at most 1280. We present some data about the census and discuss some interesting related questions.

This is joint work with Primoz Potocnik and Pablo Spiga.

Mini-courses

Overview of Magma and **Coxeter groups and complex reflection groups**

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Part 1: Introduction to the Magma language and styles of programming. Essential data structures: sets, sequences, records and associative arrays. Functions and procedures, function expressions and maps. Magma's type system and coercion. Using files. Printing and logging. The help system. These topics will be illustrated by examples drawn from a selection of Magma packages for groups, linear algebra, graph theory, number theory and algebraic geometry. The chapter on the Magma language by Geoff Bailey in the book 'Discovering Mathematics with Magma' (Spring, 2006, edited by Bosma and Cannon) will be available to workshop participants.

Part 2: Coxeter groups are implemented in Magma as permutation groups, matrix groups and finitely-presented groups. The finite Coxeter groups are fundamental to Magma's implementation of many algorithms for computing with groups of Lie type. A recent addition to Magma is a package to compute with W-graphs. As an application, the algorithms (due to Bob Howlett) provide sparse matrix representations of Hecke algebras of very large degree. The construction of reductive groups in Magma depends on the root datum of a Coxeter group. Root data can also be defined for complex reflection groups. I will describe how Magma can be used to determine all reflection subgroups of a finite complex reflection group and how this can be used to prove a recent characterisation of parabolic subgroups.

Permutation groups, small groups and subgroup structure

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Part 1: Introduction to permutation groups: We will explain the basics of permutation groups in Magma (how to construct permutations and permutation groups, finding the order, computing stabilizers, composition series, conjugacy classes, getting subgroups, etc.) We will also briefly describe the databases of permutation groups that are available (small groups, primitive groups, transitive groups).

Part 2: Subgroup structure of permutation groups and some practical examples: We will describe how the subgroup lattice of a permutation group is computed in Magma, and discuss some possible improvements, for instance to compute the subgroup lattice of the O'Nan sporadic simple group, the second Conway group, etc.

Finally, we will show some examples of applications of the topics covered in the two lectures.

Matrix groups and **Polycyclic groups**

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Part 1: We will consider techniques for basic computations with matrix groups in Magma. These include the the long-standing (Random) Schreier-Sims algorithm which constructs a stabiliser chain for a matrix group defined over a finite field. We will also discuss how to decide "geometric" properties of the matrix group in its action on its underlying vector space . Finally we will discuss the use of congruence subgroup machinery to study (infinite) matrix groups defined over infinite fields, and its use to decide such properties as the Tits Alternative.

Part 2: A standard approach to the study of a finitely-presented group is to investigate its nilpotent and soluble quotients. Powerful algorithms exist to construct power-conjugate presentations for these quotients. Such descriptions are very useful: they solve the "word problem" and are used as input to a large number of other structural algorithms. We will illustrate some of the computations involved.

Investigating finitely-presented groups

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Over the last decades, several kinds of computational methods have been developed for answering questions about groups defined in terms of generators and relations, and a large number of these have been implemented in GAP and Magma. Most of them are built on the famous 'Todd-Coxeter' procedure for enumerating the cosets of a given subgroup of finite index in a finitely-presented group, and/or the Reidemeister-Schreier procedure for finding a presentation for such a subgroup. I will describe some of these methods, including recent ones for finding all subgroups or all normal subgroups of given finite index in a finitely-presented group, and give some brief demonstrations of their implementation in Magma. If time allows, I will also mention some important contexts in which these methods can be applied.

Contributed talks

Arc-transitive graphs of order 2pq

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We present a classification of arc-transitive graphs of order twice a product of two primes. We also briefly describe the abelian normal quotient method, which works particularly well in this case.

Regular embeddings of multipartite graphs

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A 2-cell embedding of a graph into an orientable closed surface is called orientably regular (resp. regular) if its orientation-preserving automorphism group (resp. automorphism group) acts regularly on its arcs (flags). One of the central problems in topological graph theory is to classify regular maps by given underlying graphs. Let $K_m[n]$ be the multipartite graphs with m parts where each part contains n vertices. It is well known that all regular embeddings of complete graphs $K_m[1]$ and complete bipartite graphs $K_2[n]$ have been found. In this talk, we shall introduce our classification for the general case, that is, for $m \geq 3$ and $n \geq 2$. This is joint work with Mr Junyang Zhang.

One-regular graphs

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A graph is *one-regular* or *arc-regular* if its automorphism group acts sharply-transitively on the set of its ordered edges. In this report, I first review some recent results on one-regular graphs. Then I talk about an open question about the existence of arc-regular 3-valent graphs of order $4m$ for an odd integer m, which was answered by M.D.E. Conder and the author. Using the Gorenstein-Walter theorem, it is shown that any such graph must be a normal cover of a base graph, where the base graph has an arc-regular group of automorphisms that is isomorphic to a subgroup of $\mathrm{Aut}(\mathrm{PSL}(2, q))$ containing $\mathrm{PSL}(2, q)$ for some odd prime-power q. Also a construction is given for infinitely many such graphs — namely a family of Cayley graphs for the groups $\mathrm{PSL}(2, p^3)$ where p is an odd prime; the smallest of these has order 9828.

On imprimitive rank 3 permutation groups

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The rank of a permutation group on a set Ω is the number of orbits it has in its natural action on $\Omega \times \Omega$. A transitive permutation group has rank at least two, as the set of all pairs (α, α) is an orbit. The rank two groups are the 2-transitive groups, and their classification was a consequence of the Classification of Finite Simple Groups. All 2-transitive groups are primitive, but it is not necessary for a rank three group to be primitive. All primitive rank three groups have been classified. In this talk I will discuss recent work with Alice Devillers, Cai Heng Li, Geoffrey Pearce and Cheryl Praeger, investigating the imprimitive rank three groups, and motivated by a question from graph theory.

On finite groups of self-homeomorphisms of compact topological surfaces with invariant subsets

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A finite group G of self-homeomorphisms of a closed surface X (orientable or non-orientable) of topological genus $g \geq 2$ has at most $84(g-1)$ elements. On the other hand, if such a surface has $k \neq 0$ boundary components, and its algebraic genus $p = \varepsilon g + k - 1$ is greater than 1 (where $\varepsilon = 2$ or 1 according to whether the surface is orientable or not), then it has at most $12(p-1)$ self-homeomorphisms.

If such a group action allows some invariant subset of size smaller than $|G|$, then these bounds can be essentially improved. The aim of this paper is to present known results concerning this subject. We take the Nielsen-Riemann approach, which allows us to to see such groups as groups of automorphisms of surfaces with conformal structures imposed on them, and use algebraic methods based on the Riemann uniformization theorem and well-developed theory of of discrete groups of isometries of the hyperbolic plane.

Existence of non-ultra regular covering spaces whose automorphism groups are not trivial

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The study of digital covering transformation groups plays an important role in the classification of digital images. The paper deals with the problem: Is there a digital covering space which is not ultra regular but has an automorphism group which is not trivial? The present paper proves that a digital wedge has infinite fold digital covering spaces. Some of these digital covering spaces were found not to be ultra regular, while others were ultra regular. We then prove that their automorphism groups are not trivial, thus finding an answer to the problem posed above.

Group cocycles and higher representation theory

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Group cohomology is an important tool in many areas of mathematics. Examples include Quillen's work on the Adams conjecture in algebraic topology and the description of Brauer groups in class field theory. In the study of group extensions, the second and third cohomology groups play a prominent role. While the former counts the number of distinct extensions, the latter characterizes the obstruction to their existence in the first place. In this talk I will discuss recent progress on giving a representation theoretic meaning to 3-cocycles, the same way 2-cocycles are realized as projective representations. I will describe the motivation behind this construction and conclude with some open problems.

Distinguishing infinite graphs

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The distinguishing number $D(G)$ of a graph G is the least cardinal number d such that G has a labeling with d labels that is only preserved by the trivial automorphism. This talk begins with a survey of results about the distinguishing number of infinite graphs, for example the Radon graph, and graphs of higher cardinality, such as tree-like graphs, products of uncountable complete graphs, and uncountable hypercubes. Then the focus turns to the infinite motion conjecture of Tom Tucker, which asserts that connected, locally finite, infinite graphs G have distinguishing number 2 if every nontrivial automorphism of G has infinite support. For connected, locally finite graphs we show show that the conjecture is true for graphs of growth $o(n^2/(8\log_2 n)$, give examples of uncountable graphs for which it does not hold, and propose a new version of the conjecture for uncountable graphs.

Towards identification of the Majorana representations of A_{12}

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This talk contributes to the axiomatic approach to the study of the Conway–Griess–Norton algebra for the Monster sporadic simple group, known under the name *Majorana Theory*. The dimension of the Monster algebra is 196,884 and only a limited amount of direct calculations is feasible. On the other hand, the axiomatic approach has already enabled us to identify various subalgebras in the Monster algebra corresponding to small $2A$-generated subgroups in the Monster. A next goal, which appears realistic but still beyond the available technique is to identify the subalgebra corresponding to an A_{12}-subgroup in the Monster. An involution from such an A_{12}-subgroup is a $2A$-involution in the Monster if it is the product of two or six commuting transpositions, and it is a $2B$-involution if it is the product of four commuting transpositions. I believe that the linear span of the $2A$-axial vectors (also known as Majorana axes) corresponding to the $2A$-involutions in A_{12} is closed under the algebra product. An important step towards proving this conjecture is to express the $4A$-axial vectors as linear combinations of the Majorana axes. I have performed some calculations inside the centralizer of a $2B$-involution in A_{12} which show that the Majorana axes contained in the centralizer generate a codimension 1 subspace of the space generated by them together with the $4A$-axes. The calculations are conducted in the 300-dimensional Jordan subalgebra of the Monster algebra. Thus, in a sense, with every $2B$-involution in A_{12} one can associate a 1-dimensional subspace and currently I am looking for further relations among subspaces obtained in this way.

On the connectedness of the branch locus of moduli spaces of Riemann surfaces

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The moduli space of Riemann surfaces has orbifold structure, where its singular locus for genus $g \geq 3$ is the branch locus of the covering determined by the action of the mapping class group on the Teichmüller space. The singular locus is formed by Riemann surfaces admitting non trivial automorphisms. The singular locus is connected for genera 3, 4, 7, 13, 17, 19 and 59. The connected component of bigger dimension, $2g-1$, contains the hyperelliptic surfaces. For given genus $g \geq 65$, there is a connected component of complex dimension $\frac{g}{2}+2$, $\frac{g-1}{3}$ or $\frac{g-8}{3}$, according to whether the genus g is congruent to 0, 1 or 2 modulo 3 respectively.

Discrete group actions on orientable surfaces

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Lists of discrete group actions have many applications in different fields of mathematics. In combinatorics they can be used to derive lists of highly symmetrical maps of fixed genus: regular maps, vertex-transitive maps, Cayley maps or edge-transitive maps. The classification of actions of cyclic groups play the crucial rôle for enumeration problems of combinatorial objects, i.e. maps, graphs and others. Classification results can be used as an experimental material for further research.

The problem of classification of discrete actions of groups on orientable surfaces of genus $g \geq 2$ is considered. The classification of groups acting on the sphere is a classical part of crystallography. In case of torus the situation is known in general, though there are infinitely many group actions. Due to Riemann-Hurwitz equation we know that for higher genera there are just finitely many finite groups acting on a surface of a given genus. Published lists of actions go up to genus five (Broughton; Bogopolskij; Kuribayashi and Kimura). For small genera, the the classification can be done with help of computer algebra systems. Using Magma we derived the list of actions of discrete groups on surfaces of genus $2 \leq g \leq 24$.

Chord properties in Euclidean geometry

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First, we study curves in a Euclidean space of arbitrary dimension such that the chord joining any two points on the curve meets it at the same angle. Next, we study hypersurfaces in a Euclidean space of arbitrary dimension such that the chord joining any two points on the hypersurface meets it at the same angle. As a result, we give a complete characterization of such curves (resp. hypersurfaces) in Euclidean space with arbitrary dimension.

This is joint work with Dong-Soo Kim (*Department of Mathematics, Chonnam National University, Kwangju 500, Republic of Korea*), `dosokim@jnu.ac.kr`.

Quasi m-Cayley strongly regular graphs

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A graph Γ is called a *quasi m-Cayley graph on a group G* if there exists a vertex $\infty \in V(\Gamma)$ and a subgroup G of the vertex stabilizer $\mathrm{Aut}(\Gamma)_\infty$ of the vertex ∞ in the full automorphism group $\mathrm{Aut}(\Gamma)$ of Γ, such that G acts semiregularly on $V(\Gamma)\setminus\{\infty\}$ with m orbits.

In this talk I will present recent results about strongly regularity of such graphs.

This is joint work with Luis Martinez Fernandez, Aleksander Malnič and Dragan Marušič.

Classification of some regular Cayley maps on dihedral groups

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In this talk, the classification of regular Cayley maps on dihedral groups with odd valency will be given. We also consider reflexible regular Cayley maps on dihedral groups. This is joint work with István Kovács.

On the Wilson operators

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The jth hole operator H_j creates a new map $\mathcal{M}^{H_j}$ from an oriented map $\mathcal{M}$ such that the faces of $\mathcal{M}^{H_j}$ are the jth holes of $\mathcal{M}$, that is, cyclic sequences of edges, each two consecutive sharing a vertex, so that at each vertex, the adjacent edges subtend j faces on the right. The map $\mathcal{M}^{H_j}$ is well defined whenever the valency of the map $\mathcal{M}$ is coprime to j. These were introduced by Steve Wilson in 1979.

The operator H_{-1} has always been regarded as producing the mirror image, and when restricted to the class of n-valent maps, H_j has been identified with the operator $H_{j\,(\mathrm{mod}\,n)}$. But ... is this really the case?

On the split structure of lifted groups, I

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Let $p\colon \tilde{X} \to X$ be a regular covering projection of connected graphs, and let CT_p denote the group of covering transformations. The problem whether a given group of automorphisms $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$ lifts along p as a split extension of CT_p by G is analysed in detail in the case when CT_p is abelian. The analysis is done without actual inspection of the covering graph or the lifted group, that is, using just the encoded information on the base graph X via Cayley voltages. We also consider the special case when some complement of CT_p within $\tilde{G}$ has an orbit which is a section over a G-invariant set of vertices of X. This is joint work with Rok Požar.

Some vertex transitive Integral graphs

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An *integral* graph is a graph for which all eigenvalues are integers. The *spectrum* of a graph is the eigenvalues with their multiplicity. In this talk, we take previous results that give a list of spectra that are possible for a graph that is connected, 4-regular, bipartite, and integral; and consider the subgraph configurations necessary to eliminate spectra that cannot be realized by a vertex-transitive graph. Additionally, we determine that some of these integral spectra are realized by Cayley graphs.

This is joint work with Ian M. Wanless (ian.wanless@monash.edu).

A remark on the computation of normalizers of intransitive permutation groups in symmetric groups

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I have been studying automorphisms of coherent configurations. They are related to the normalizers of permutation groups. I applied the automorphism group to speed up the computation of normalizers, if it is formed by the group. The argument works if the group and its normalizer have a common orbit, for instance, if it is transitive. Here I will give a remark to speed up the computation of normalizers of intransitive groups. We consider the following kind of groups as an example. A group G has a few orbits and the actions of G on the orbits are isomorphic mutually as permutation groups.

It is announced that the computation of normalizers and subgroup conjugacy was greatly improved in Magma V2.18 on 10th December 2011.

A new variational principle for discrete integrable systems

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The theory of Discrete Integrable Systems (DIS) has developed in the last two decades into a major strand of research on the interface of pure and applied mathematics and mathematical physics. A particular aspect is the study of exactly integrable partial and ordinary difference equations and their connection with incidence theorems and difference geometry (‘discrete differential geometry’).

In the present talk I will focus on a particular recent development related to the variational calculus associated with such systems, which are characterized by the property of *multidimensional consistency*. The novel idea of *Lagrangian multiform structures*, proposed by S. Lobb and the speaker in 2009, allows the encoding of the latter property into a least-action principle framework applied to both the dependent variables as well as with respect to the geometry in the space of independent variables. Examples of this structure for integrable systems in one, two and three dimensions are given as well as a universal action for affine-linear quadrilateral lattice equations belonging to the so-called ABS list of integrable partial difference equations on the two-dimensional lattice.

GI-graphs and their groups

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We consider the class of GI-graphs, generalising the class of generalised Petersen graphs. The GI-graph $GI(n; j_1, j_2, ..., j_t)$ is a $(t+1)$-valent graph on the vertex set $\mathbb{Z}_t \times \mathbb{Z}_n$ with edges of two kinds:

- edges from (s, v) to (s', v) for all distinct $s, s' \in \mathbb{Z}_t$, for every $v \in \mathbb{Z}_n$,
- edges from (s, v) to $(s, v + j_s)$ and $(s, v - j_s)$ for all s and v.

We study different properties of GI-graphs. By classifying different kinds of automorphisms, we try to describe for each GI-graph its group of automorphisms, and determine which GI-graphs are vertex-transitive. They may be edge-transitive only for $t \leq 3$ or equivalently, for valence less or equal to 4.

This is joint work in progress with Marston Conder and Arjana Žitnik.

On the split structure of lifted groups, II

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Let $p\colon \tilde{X} \to X$ be a regular covering projection of connected graphs, and let CT_p denote the group of covering transformations. We present an algorithm for testing whether a given group of automorphisms $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$ lifts along p as a split extension of CT_p by G in the case when CT_p is abelian. Next, we present an algorithm for testing whether G lifts as a split extension such that some complement of CT_p within $\tilde{G}$ has an orbit which is a section over a G-invariant set of vertices of X. Both algorithms are part of a larger package (implemented in Magma) for doing computations with graph covers. The implementation allows computing with graphs having semi-edges.

Codes from lattice and related graphs, and permutation decoding

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Codes of length n^2 and dimension $2n-1$ or $2n-2$ over the field $\mathbb{F}_p$, for any prime p, that can be obtained from designs associated with the complete bipartite graph $K_{n,n}$ and its line graph, the lattice graph, are examined. The parameters of the codes for all primes are obtained and PD-sets are found for full permutation decoding for all integers $n \geq 3$.

This is joint work with J.D. Key.

The diameter of permutation groups

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The *diameter* of a finite group G is the maximum of the diameters of all Cayley graphs $\Gamma(G, S)$, where S is a set of generators for G closed under inverses. The diameter can be proportional to $|G|$ (for example in cyclic groups or in groups with large cyclic factor groups), but a conjecture of Babai states that for simple G the diameter is a polylogarithmic, $(\log |G|)^{O(1)}$ function of $|G|$.

Starting with the seminal work of Helfgott in 2008 for the groups $\mathrm{PSL}(2, p)$, p prime, Babai's conjecture became a very active area of asymptotic group theory. For Lie-type groups, the current status is that the conjecture is true for groups of bounded rank (Pyber, Szabó and Breillard, Green, Tao, 2011). However, the techniques developed for Lie-type groups are not applicable to the central case of alternating groups.

For the diameter of alternating groups A_n, Babai's conjecture requires a $n^{O(1)}$ bound. We can prove a somewhat weaker, quasipolynomial bound:

$$\mathrm{diam}(A_n) < \exp((O(\log n)^4 \log\log n)).$$

The same bound holds for the diameter of all transitive groups $G \leq S_n$.

(This is joint work with Harald Helfgott.)

The quantitative characterization of all finite simple groups

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Let G be a finite group and let $\pi_e(G) = \{o(g) \mid g \in G\}$, that is, the set of all element orders for G. In 1987, we put forward the following conjecture:

Conjecture. All finite simple groups G can determined uniformly using their orders $|G|$ and their element orders $\pi_e(G)$.

This conjecture is proved recently. In this talk we present a historical survey and pose some related questions.

Imprimitivity of locally finite, 1-ended, planar graphs

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Using results from group theory, we offer a concise proof of the imprimitivity of locally finite, vertex-transitive, 1-ended planar graphs, a result previously established by J E. Graver and M.E. Watkins (2004) using graph-theoretical methods. (This is joint work with Mark E. Watkins (Syracuse University).)

2-transitivity in finite circle geometries

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Circle planes or Benz planes are incidence geometries with points and circles, which are subsets of the point set and contain at least three points. They axiomatize the geometries of non-trivial plane sections of a quadratic set (ovoid, oval cone or ruled quadric) in 3-dimensional projective space over a field. There are only a few models known for finite circle planes. It is a long-standing open problem whether or not the known models are the only ones there are. The classical models of Möbius, Laguerre and Minkowski planes, obtained from an elliptic quadric, elliptic cone or a ruled quadric, respectively, are highly homogeneous and exhibit some kind of triple transitivity.

In this talk I discuss what can be said if the homogeneity assumptions are reduced to 2-transitivity. For elation Laguerre planes this becomes a question about modular representations of certain groups of Lie type.

Distinguishing maps: angles, cliques and transpositions

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We classify all finite maps M with distinguishing number $D > 2$, that is, maps such that each set of vertices has non-trivial stabilizer in $\mathrm{Aut}(M)$. The orientation-preserving case $\mathrm{Aut}^+(M)$ was covered by the speaker in *Elec. J. Comb* (2011); there are only 5 such maps having no vertices of valence 1 or 2. The general case is much more complicated; there are 32 possible underlying graphs and most are associated with many different maps, often because of Petrie duality. The list of maps includes several unfamiliar Cayley maps with intricate symmetries. The methods are almost completely *sui generis* and depend deeply on the new concept of angle measure for a map. In turn, angle measure has led to an elegant proof that the clique number of a regular (reflexible) map must be $2, 3, 4, 6$. Another off-shoot has been a rediscovery of a folk theorem about generating the symmetric group by transpositions.

Autotopisms of Latin squares

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A *latin square* is a matrix in which each row and column is a permutation of the same set of symbols. There is a natural action of $S_n \times S_n \times S_n$ on latin squares of order n, where the three permutations permute the rows, the columns and the symbols respectively. The stabiliser of a latin square L under this action is its *autotopism group*, denoted $\mathrm{atp}(L)$. We consider the question of which elements of $S_n \times S_n \times S_n$ belong to $\mathrm{atp}(L)$ for some L. We show a number of general results which together are powerful enough to answer the question for all $n \leq 17$, with the exception of one ad-hoc case.

Towards a geometry of operads: dendroidal polytopes

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The study of operads is intimately related to trees since the latter govern the combinatorics of the compositions of multivariable functions. This specific tree combinatorics is captured by means of the dendroidal category Ω giving rise to dendroidal sets which are expected to posses a geometric realization. For a restricted class of dendroidal sets, known as simplicial sets, that correspond to linear trees, a geometric realization is well known and rests on the simple observation that linear trees correspond to certain polytopes: the standard n-simplices. In this talk I will give an order theoretic definition of the dendroidal category Ω, show that the poset of subobjects of a tree $T \in \Omega$ is very nearly an abstract polytope, christen these new objects *abstract dendroidal polytopes*, and hint at the definition of concrete metric objects to serve as realizations of them.

Tri-circulant edge-transitive tetravalent graphs

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This talk will outline the problem of determining all edge-transitive graphs on $3n$ vertices admitting a symmetry which acts on the vertices in three cycles of length n. We show the nine feasible diagrams and show how considerations of toroidal maps can give us an easy elimination of four of the nine. Two others have a more difficult elimination whose difficulties we will ignore. We end by showing nearly proven conjectures about the remaining three diagrams.

Galois actions on regular dessins of small genera

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According to ideas of Belyi and Grothendieck, maps and hypermaps (or 'dessins') induce on oriented surfaces always a unique conformal structure, moreover, even a structure of a smooth projective algebraic curve defined over a number field. Can we determine these number fields explicitly and understand the action of the absolute Galois group on these dessins? What does this link between arithmetic and graph embeddings look like?

This talk gives a report on methods and results concerning these questions for all regular dessins of genera up to 18. (This is joint work with Marston Conder, Gareth Jones, Manfred Streit, to appear in *Revista Matemática Iberoamericana.*)

On vertex-transitive non-Cayley graphs

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In this report, we shall give some results on vertex-transitive non-Cayley graphs with special orders and with small valency.

Gossiping and routing in second-kind Frobenius graphs

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Two kinds of Cayley graphs on Frobenius groups exhibit interesting gossiping and routing properties, making them attractive candidates for modelling interconnection networks. In this talk we will focus on second-kind Frobenius graphs. In the case when the kernel of the Frobenius group involved is abelian of odd order, we find the exact value of the minimum gossiping time for such a graph under the store-and-forward, all-port and full-duplex model, and prove that the graph admits optimal gossiping schemes with fine properties. In the case when the kernel of the Frobenius group is abelian of even order, we give an upper bound on the minimum gossiping time under the same model. As examples we will discuss a family of second-kind Frobenius graphs which contains all Paley graphs and connected generalized Paley graphs.
