Mathematicians and light bulbs

At the ANZIAM 2005 conference in Napier, a competition was held on the subject "How many mathematicians does it take to change a light bulb?". This Miniature contains the text of the winning entry with names changed to protect the identity of the mathematicians involved.

I feel I need to justify my lack of seriousness in writing about a subject to which Mathematics is not traditionally applied. More than 30 years ago, Martin Gardner announced a number of plausible but untrue results in his regular column in Scientific American, as what was later realised to be an April Fools joke. This will probably be the last Miniature I write for an April issue so, ever the rebel, this will be my last chance to say that if was good enough for Martin Gardner it is good enough for me.

So I return to the light bulb question and I announce the answer and the steps which led to this conclusion.

The answer is $|1|$, under certain conditions, otherwise it is γ.

Before our group of mathematicians could tackle this question, we felt we needed some theory. Gav Mehrtens wanted some axioms and a definition of *light bulb*. Mast Candour thought that it wasn't an interesting question: better to ask whether the number being sought was actually integer, and if not whether it was rational, and if not whether it was algebraic. Jam Snyder, showed his contempt for this line of discussion by asking, sarcastically, whether the number was even finite.

Axian Ford, thought we shouldn't hold back from using knowledge from Physics and that finiteness didn't matter too much because we could always extract the part of the result that was observable by applying renormalization, or some other such trick.

Jon Harpoon thought we were on quite the wrong track and that we needed a model. A considerable time was spent trying to agree on a model that everyone was happy with. When we realised we weren't going to get any agreement on the model, we decided to follow a lead from Grim Hwayk and considered first the question:

"Could a single mathematicians change a light bulb?"

We all thought not, but how about a second attempt, in which the first mathematician is now joined by a second? If the answer had been yes, we still wouldn't have enough for a publishable paper so we decided to assume, for the moment, that the answer was "No", or possibly "Maybe".

Because we couldn't be really sure about this step, we decided to assign a probability to it. In the absence of any reliable evidence we set the probability that this second attempt would be successful as $\frac{1}{2}$. There were now two cases known to the more general question:

"What is the probability that success will be achieved by n or fewer mathematicians?"

Both known cases fitted into the formula

$$P_n = \frac{n-1}{n},$$

so this was assumed to be the answer for the general case.

We really wanted the value of p_n, the probability that *exactly* n mathematicians would be needed.

Davier Johannes offered to help with the calculations, which now involved probability and conditional probability. He came up with the formula

$$p_n = \begin{cases} 0, & n = 1, \\ \frac{1}{n(n-1)}, & n > 1. \end{cases}$$

The next step was to find the expected value of n and he was also able to give this result:

$$E(n) = \sum_{k=2}^{\infty} k \cdot \frac{1}{k(k-1)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots .$$

Jam Snyder now felt vindicated by his earlier scepticism, because this sum was infinite. Axian Ford also could now make a specific proposal, coming out of his experiemce as a Mathematical Physicist, and this was to subtract from $\sum_{k=1}^{n} k^{-1}$ an adjustment of $\ln(n)$, so that the limit was the Euler constant, given by

$$\gamma = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \ln(n) \right).$$

Mast Candour tried to determine whether this was rational or not but soon gave up. Gav Mehrtens, ever the logical conservative, now questioned the very first step. What if a single mathematician *had* been able to accomplish the task alone? We decided to offer an alternative answer to cover this case. However, there was a short debate as to why 1 can be assumed to be real and positive. To cover our backs we decided to write $|1|$.

It was generally agreed that offering two answers, when we weren't really sure which was correct, was a mark of intellectual integrity. Furthermore, it offered us the opportunity of a second paper later, with a guaranteed citation of the first paper, and altogether a firm case for continued funding.

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