Punishing horse thieves and game theory

According to George Savile, Marquess of Halifax, Men are not hanged for stealing horses but that horses may not be stolen. This sounds good to me: a punishment so severe that it is never imposed.

Sometimes interesting mathematics comes from reading newspapers. Occasionally the connection is direct but more often, in my experience, it is possible to theorise about the opposite to what is printed. A few days ago someone wrote a letter to the *NZ Herald* stating that self-interest was always the best course of action and that, indeed, any other choice was immoral because of being self-destructive. An authority by the name of Ayn Rand, author and advocate of laissez faire economics, was referred to. I am not sure about the morality issue but it is possible to model some situations in which self interest can be contrasted with magnanimity and at least study the consequences of individual choices.

I would like to look at a very simple multi-player game characterised by symmetry amongst the players and the existence of a Nash equilibrium in which every player acts with complete self-interest. If "Nanny state" wanted to tax antisocial behaviour, it might be possible to do this in such a way that no revenue is collected at all and that the threat of redistribution of resources is enough to make actual redistribution unnecessary.

Suppose there are $n+1$ players who can independently choose either **0** or **1**. We will think of **0** as representing self-interest because the reward will be greater for this than for the other choice no matter what choices the other players decide on. In game theoretic language the pay-off will be denoted by u_i for each player choosing **0** assuming that a total of i players choose **1**. On the other hand, the payoff will be denoted by v_{i-1} for players choosing **1** assuming that i players make this choice. To make self-interest always advantageous, we need to assume that $u_i > v_i$ for $i = 0, 1, 2, \dots, n$. We will also assume that the sequences $[u_0, u_1, \dots, u_n]$ and $[v_0, v_1, \dots, v_n]$ are each strictly increasing, and that $v_n > u_0$. This means that there is a genuine conflict between acting according to self interest in which the other choice would always be disadvantageous, and acting magnanimously which would be better, if everybody else acted in the same way.

This model can be used to describe the choice between driving a car to work versus using public transport. If many commuters are in the same position, then $-u_i$ would represent the travel costs in driving, assuming that i people use public transport, and $-v_i$ would represent the cost of using public transport assuming that i other people are doing the same. The assumption that the u and v sequences are increasing corresponds to the observation that roads get more and more clogged the more cars are on them.

The same ideology that places individualism on a high moral plane rejects the idea of state interference as being contrary to the efficient functioning and the general well-being of society. In terms of the model game I have described, this need not be the case.

Suppose a tax is imposed on the players choosing **0**, not for revenue but to subsidise the players choosing **1**. In the transport example, this would correspond to part of the cost of public transport being paid by those who do not actually use it. If the subsidy is designed to bring the payoff up to v_n then the cost shared amongst the players choosing **0** would need to be $i(v_n - v_{i-1})/(n+1-i)$ per player. Thus the adjusted payoff for these $n+1-i$ players would be $u_i - i(v_n - v_{i-1})/(n+1-i)$.

Suppose a referendum was being held amongst all players to decide if this tax should be imposed. How should one vote? The answer is that everyone should vote for the tax because everyone will be better off if it is imposed.

Another interesting game involves players numbered $1, 2, \dots, n$. For player P, P^+ will denote player number $P+1$ (or number 1 if $P = n$). Each player can choose to play **0** resulting in a payment of 1 to P, or **1** resulting in no payment to P but a payment of 2 to P^+, in addition to the 1 P^+ would receive if P^+ chooses **0**. As an example, if $n = 3$, then the payoffs will be given by the following table in the eight possible cases.

Play made by player 1	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**1**
Payoff to player 1	1	0	1	0	3	2	3	2
Play made by player 2	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**
Payoff to player 2	1	3	0	2	1	3	0	2
Play made by player 3	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**
Payoff to player 3	1	1	3	3	0	0	2	2

In the case $n = 2$, this game is an example of the "Prisoner's Dilemma" and, for the iterated version of this game, the tit-for-tat response to uncooperative treatment by your opponent is regarded as the way to bring this player into line. If $n > 2$, things are not so simple because P would have to punish P^+, if this is needed, on behalf of P^{++} and any eventual benefit flowing back to P is remote.

In real life, there is no such simple circular structure but a player P will from time to time recognise the existence of a P^+ and will have an opportunity to treat P^+ well, without any clearcut reciprocal advantage. In ethical systems in which altruism plays a part, only a belief that kind actions are intrinsically good and that selfish actions are bad will guide a player to choose **1** rather than **0**. If the whole of society thinks the same way and is willing to conform, then everyone benefits. Acting outside the standards of this society might not lead to the fate prescribed for horse thieves, but it can lead to censure strong enough to amount almost to the same thing.

John Butcher, `butcher@math.auckland.ac.nz`