Mathematics and Music

It is often said that mathematics and music go together. Although many mathematicians take a serious interest in music, and for that matter in other cultural pursuits, it doesn't always work the other way round. From my observations, I would say that it is quite rare for a musician to take a genuine interest in mathematics or, for that matter, in any science.

Today I want to write about two young mathematicians who have added enormously to my positive feelings about mathematics in New Zealand. They have in common also a great love of music.

One of the really great things about mathematics in this country is the annual colloquium. At least in its early days in the sixties, almost every mathematician in the country took part. The interests of the participants were remarkably diverse for such a small country but everyone seemed to get something out of being part of this annual gathering. Even though some of the early enthusiasm has now fizzled out, the days of greatness are far from over.

I didn't realise that Aroon Parshotam had a secondary interest in music until I heard him speak at the 2001 New Zealand Mathematical Colloquium in Palmerston North. He spoke on "Music as Applied Mathematics in Action", rather than on a topic arising from his professional work for the Landcare Research Institute. I believe that Aroon is all the better as a practical mathematician because of his many other interests, including music. Aroon is effective in his work not only because of his knowledge and training and experience but also because of his personality. He can talk sympathetically to anyone about anything. I believe he is especially effective as a mathematical scientist because he can engage with a potential client without allowing his specialist knowledge to become a barrier between them. The mathematical sciences need people like him – people with the humanity associated with a love of music.

I have been privileged to have been one of the teachers of Ruby Chen, who like Aroon combines interests in mathematics and music. Ruby had her primary education in Taiwan; she told me that music was not given much importance in the education system there, because it is not perceived to be utilitarian in the sense that science and mathematics are. In fact classes scheduled for musical appreciation often drifted into other arguably more practical topics.

When Ruby was undertaking her secondary schooling in New Zealand she felt there was a better balance and she actively enjoyed both music and mathematics as parts of her education. She has become an accomplished exponent of the Gu Zheng, as well as of Western instruments. While doing her music degree, Ruby enrolled for a single paper in Applied Mathematics. She became fascinated again with this other great love of her intellectual life. Most especially, the idea of mathematical modelling impressed itself upon her. From this starting point she went on to complete a science degree to place alongside her music degree. Eventually, she completed an MSc thesis while still pursuing a career as a performing musician.

At the present time, Ruby is active as a musician more than as a mathematician. Does this mean we have lost her to mathematics now that her formal studies have been completed? Even though she may never have a career in mathematics, although I hope she does, this is hardly the point. The culture of mathematics is now part of her culture just as the culture of music will always be part of Aroon's. I once asked Ruby which of Mathematics and Music is the more important to her. She replied that they are both part of her life and it is difficult to say where one starts and the other ends. I suspect that Aroon would give a similar answer.

For x a positive real, but not an integer, let

$$\phi(x) = \frac{[x]x + 1}{x - [x]},$$

where $[x]$ is the integer part of x. For given x_0, define $x_1 = \phi(x_0)$, $x_2 = \phi(x_1)$, and so on. The sequence possibly terminates if for some n, x_n is an integer.

1. Is it true that the sequence terminates if and only if x_0 is rational?
2. Are the members of the sequence monotonically increasing and, if x_0 is irrational, is the sequence unbounded?
3. Is there any conceivable application of this sequence?

I turn 70 on the day I submit this miniature to the editor. When I first volunteered to write this one-page article three times a year I was looking for something to keep me busy during my leisurely retirement. There have been some spin-offs. The then editor of the New Zealand Mathematics Magazine asked if he might reprint one of my pieces and I decided it might be better re-written for a different readership. Thus began the Mathematical Apologies that I now write regularly for the Magazine. A few people overseas have told me they have discovered the Miniatures on my webpage and sometimes read them.

I am now starting to wonder if I have carried on with these enterprises long enough. My time in retirement is far from leisurely but I really enjoy writing these small jottings. My personal enjoyment alone, however, hardly justifies the effort. I would value comments from other people before deciding if I should keep on writing these one-page articles.

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